The Wheels on the Bus

I. UNIT OVERVIEW & PURPOSE:

This unit will allow students to use their knowledge of proportional reasoning, specifically scale factor and scale drawing, and direct variation to devise an alternate school bus transportation proposal for the high school in our county.

II. UNIT AUTHOR:

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III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry, Measurement, Ratios & Proportions, Direct Variation, Logic & Reasoning, Statistics

V. OBJECTIVES:

The student will use standard measurement tools in combination with county maps of state-maintained roads and ratios and proportions to determine the distance of existing and proposed bus routes, calculate the actual dollar amounts of existing budget allocations, calculate the actual cost of existing bus routes and actual cost of proposed bus routes, and calculate any savings between existing and proposed bus routes. The students will explain and justify their findings and present their proposals in a formal paper and a tri-fold panel.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE 1: The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 7: Use similar geometric objects in two- or three- dimensions to
- a) compare ratios between side lengths, perimeters, areas, and volumes;
- d) solve real-world problems about similar geometric objects.

MPE 8: Compare distributions of two or more univariate datasets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

MPE 9: Design and conduct an experiment / survey. Key concepts include:
- d) data collection
- e) data analysis and recording

MPE 12: The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will

select and use appropriate representations for analysis, interpretation, and prediction.

VII. CONTENT:

Students will use mathematics to collect, interpret, and analyze data, calculate using scale factor from a scale drawing, in a manner similar to models such as those found in the fields of computer software technology, architecture, and logistical planning. Students will utilize the computer labs, the internet, and guest speakers as resources to find real life data.

VIII. REFERENCE/RESOURCE MATERIALS:

- County transportation official
- Computer labs
- County maps of state maintained roads

IX. PRIMARY ASSESSMENT STRATEGIES:

Assessment will be through scoring of students' existing transportation cost calculations, proposed transportation cost calculations, formal proposal paper with proposed savings, and a tri-fold panel.

X. EVALUATION CRITERIA:

Rubrics for existing cost, proposed cost, final proposal paper, and tri-fold panel are included in each lesson plan. The expectation for the final proposal paper and tri-fold panel are also included.

XI. INSTRUCTIONAL TIME:

Four 90-minute blocks. There is an extended unit component which includes presenting at a math fair, and/or conducting further research into best time for high schools and amount of sleep needed by high-school aged students.

Lesson 1: Organize the Chaos

Strand
Geometry: Ratios & Proportions, Scale Factor (Scale Drawing)

Mathematical Objective(s)

- Students will solve proportions.
- Students will perform calculations using a given scale factor in a real life application.
- Students will convert percentage and calculate dollar amounts.
- Students will analyze existing transportation costs, create research questions, and make preliminary predictions for alternatives to reducing transportation costs.

College and Career Ready Mathematics Performance Expectation

MPE 7: Use similar geometric objects in two- or three- dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;
d) solve real-world problems about similar geometric objects.

RELATED SOL

G.14A

NCTM Standards

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- apply appropriate techniques, tools, and formulas to determine measurements.
- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- make decisions about units and scales that are appropriate for problem situations involving measurement

Additional Objectives for Student Learning

- Students will research and analyze existing transportation costs, and propose alternatives to reduce transportation costs.
- Students will conduct a formal presentation to invited county officials at a math fair.

Materials/Resources

- Classroom set of calculators
- Pie chart showing county allocated educational expenses

Assumption of Prior Knowledge

- It is assumed that students have worked with scale factor and solved proportions in previous math classes.
- Relevant contexts that are drawn on in relation to this concept include map reading /design and logistics planning.

Introduction: Setting Up the Mathematical Task

- Goal of the lesson.
  - "In this lesson, students will practice solving proportions, analyze existing transportation costs, collaborate while determining questions that need to be answered and information that needs to be gathered to generate a proposal, and reflect on the current status of a proposal."

Timeline for lesson: One 90 minute block

What	Who	Amount of Time
Warm up	Students will complete five proportions.	15 minutes
Introduction / discussion about why students will be asked to create an alternate transportation proposal	Whole class	15 minutes
Students will form groups of two or three or work individually on the proposal	Students	5 minutes
Collaborate / Brainstorm on what types of information will be needed to initiate the project	Students (Individual or groups of two or three)	15 minutes
Discussion of brainstorming ideas (Teacher to guide to necessary topics not addressed in brainstorming) – Create KWL chart	Students and Teacher	10 minutes 15 minutes
Journal Reflection	Students	10 minutes

- Introduction Discussion
  - Discussion lead-in: "Our county faces budget challenges just like any business or county. Many of our buses come to school only partially full giving the appearance that the current transportation plan is not as cost efficient as it may be. You will be making decisions on how to reduce the educational transportation costs for our high school." (Note: If your county has more than one high school then use maps of state-maintained roads that feed your school.)

- Display the pie chart of the county allocated educational expenses. Have students calculate the dollar equivalent for each sector. If a published pie chart does not exist, then have the students develop one using the current budget expenditures.
- Students will focus on the transportation sector and answer the following questions:
    - How many of you ride the bus to and/or from school?
    - How many of you drive to school or have someone drive you to school?
    - If you ride the bus, is your bus full?
    - Are the current busses and routes for our school being used as efficiently as possible?

- Creation of KWL chart (summary of brainstorming session)

K (What is known)	W (What we want to learn)	L (What we did learn)

- Journal Reflection: Students will record individually their initial thoughts about what they think their proposals might be. (Students will be referring to these later.)

Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
    - This activity should address the needs of kinesthetic, auditory and visual learners.
    - Adjusting groups to ensure students with processing, memory, or motor issues will have their needs met
    - English language learners (ELLs); ELL students may need additional time to translate the directions and outcomes.
- Provide a pie chart for each child with processing or visual challenges.

For Next Class

- Confirm that a representative from the department of transportation will attend and have questions ready for him/her from brainstorming session.

- Sample questions: How many busses are used for the high school run? How many high school students will a bus hold? What is the average cost of diesel fuel per gallon? What is the average miles per gallon for one of our school busses? What is the yearly average cost of bus maintenance? What is an average bus driver's salary? What is the average insurance cost for a bus? What are our county's legal obligations for transporting students to school on school busses? How many gallons of fuel does one of our bus tanks hold? What is the average length of time a student rides the bus? What are the existing bus routes for the high school? How many students are in the high school? How many students have purchased parking passes? What is the breakdown of students (i.e. number of freshman, sophomores, juniors, seniors)? How many students are planned for on each route?

- If there is no transportation department representative coming, teacher must have answers to questions from brainstorming session.

- As an alternative, reserve computer lab for students to research answers to their brainstorming questions.

Warm up: Solve each of the following proportions. Show appropriate work.

1. $\frac{4}{15} = \frac{w}{75}$

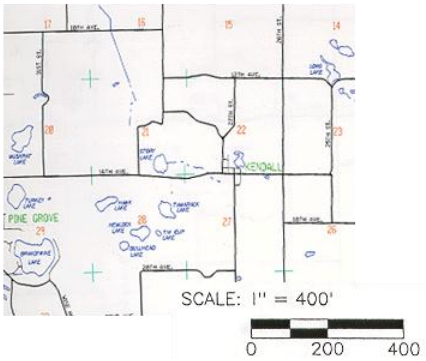
2. $\frac{6}{10} = \frac{15}{n}$

3. $\frac{p}{36} = \frac{9}{8}$

4. 75 ml of olive oil will make 15 entrees. If a chef has 300 ml of olive oil how many entrees can he make?

5. Using the following map scale:

What is the distance in feet between two random lakes if they measure 5 inches apart? (The actual location of the lakes is inconsequential; rather, students will use the map scale).



Solutions to warm up:

1. w = 20 2. n = 25 3. p = 40.5 4. x = 60 entrees 5. x = 2000 ft.

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Mr. Michael Bolling, Virginia Department of Education

Lesson 2: Answers, Code, and Calculate

Strand

Geometry - Measurement, Ratios & Proportions (Scale Factor), Direct Variation, Statistics

Mathematical Objective(s)

- Students will solve proportions.
- Students will measure using a measuring tool.
- Students will gather and organize information to their research questions provided by a guest speaker (or by other means from lesson 1).
- Students will perform calculations using a given scale factor in a real life application, specifically the distance of roads using a map key.
- Students will calculate a variety of daily school bus transportation costs and submit results for assessment (grading by rubric).
- Students will research and analyze existing transportation costs, and propose alternatives to reduce transportation costs.

Virginia College and Career Ready Mathematics Performance Expectation(s)

MPE 1: The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 7: Use similar geometric objects in two- or three- dimensions to
b) compare ratios between side lengths, perimeters, areas, and volumes;
d) solve real-world problems about similar geometric objects.

MPE 8: Compare distributions of two or more univariate datasets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

MPE 9: Design and conduct an experiment / survey. Key concepts include:
f) data collection
g) data analysis and recording

MPE 12: The student will transfer between and analyze multiple representations of

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functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

Related SOL

A.4f, A.8, AII.10, G.14A

NCTM Standards

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- apply appropriate techniques, tools, and formulas to determine measurements.
- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use geometric models to gain insights into, and answer questions in, other areas of mathematics
- make decisions about units and scales that are appropriate for problem situations involving measurement

Materials/Resources

- County maps displaying all state maintained roads
- Highlighters, markers, crayons, etc.
- Classroom set of rulers
- Classroom set of calculators
- Representative from the department of transportation for the county
- Possible access to personal computers (if no speaker)

Assumption of Prior Knowledge

- It is assumed that students have studied similar figures and solved proportions in previous math classes.
- It is assumed that students can calculate averages and apply results to reasonable conclusions in real life applications.
- Students should be able to use the scale factor of a scale drawing.
- Related skills that will be needed include map reading/design and logistics planning.

- **Introduction: Setting Up the Mathematical Task**
  - Goal of the lesson:

    "In this lesson, students will be provided with the answers to their research questions, organize and analyze this data, calculate the various transportation costs per bus for the high school, and calculate the total transportation cost for the high school."

Timeline for lesson— Two 90-minute blocks:

What	**Who**	**Amount of Time**
Guest speaker will provide answers to students' research questions	Guest speaker	30 minutes
Students will work individually or with chosen groups to code the county maps with existing bus routes (i.e. color code for each bus/existing route)	Students	30 minutes
Groups or individuals will use data to calculate existing transportation costs per bus per day and arrive at a total cost per bus per day	Students	60 minutes (30 minutes into the next day)
Groups or individuals will devise new routes or alternative transportation methods to arrive at a reduction in cost per bus (students will refer to journal writing from lesson 1)	Students	60 minutes
Students will submit their transportation cost calculations for existing bus routes.	Students	Closure of 2nd part of lesson 2 (day 3)

Assessment of existing transportation calculations:

	Coding of existing bus routes	Mileage distance calculations	Fuel cost calculations	Calculation of Average cost per day of Annual Costs	Totals per day per bus	Grand total for high school
4	Legible, clearly defined, includes a key	Calculations are 93 to 100% accurate	Calculations are 93 to 100% accurate	Calculations are 93 to 100% accurate	Calculations are 93 to 100% accurate	Calculations are 93 to 100% accurate
3	Legible, somewhat defined, includes a key	Calculations are 85 to 92% accurate	Calculations are 85 to 92% accurate	Calculations are 85 to 92% accurate	Calculations are 85 to 92% accurate	Calculations are 85 to 92% accurate
2	Legible, somewhat defined, no key	Calculations are 77 to 84% accurate	Calculations are 77 to 84% accurate	Calculations are 77 to 84% accurate	Calculations are 77 to 84% accurate	Calculations are 77 to 84% accurate
1	Legible, not clearly defined, no key	Calculations are 69 to 76% accurate	Calculations are 69 to 76% accurate	Calculations are 69 to 76% accurate	Calculations are 69 to 76% accurate	Calculations are 69 to 76% accurate
0	Not legible, not clearly defined, no key	Calculations are 68% or less accurate	Calculations are 68% or less accurate	Calculations are 68% or less accurate	Calculations are 68% or less accurate	Calculations are 68% or less accurate

Strategies for Differentiation

- The various materials will address the needs of a diverse population of students such as kinesthetic, auditory, and visual learners.
- There is an opportunity for peer tutoring.
- English language learners (ELLs)—some of the vocabulary may take further explanation.
- All students will have the opportunity to create coding techniques according to their level of expertise.
- High-ability students will have the opportunity to move more rapidly into their proposals for cost reduction.

Lesson 3: The New Wheels on the Bus!

Strand
Geometry – Measurement, Ratios & Proportions, Deductive Reasoning, Statistics

Mathematical Objective(s)

- Students will solve proportions.
- Students will measure with a measuring tool.
- Students will perform calculations using a scale factor.
- Students will formulate an alternative cost reducing measure for high school transportation costs, including logistical planning.

College and Career Ready Mathematics Performance Expectation

MPE 1: The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 7: Use similar geometric objects in two- or three- dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;
d) solve real-world problems about similar geometric objects.

MPE 8: Compare distributions of two or more univariate datasets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

MPE 9: Design and conduct an experiment / survey. Key concepts include:
d) data collection
e) data analysis and recording

MPE 12: The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

RELATED SOL

A.4F, A.8, AII.10, G.14A

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NCTM Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Build new mathematical knowledge through problem solving;
- Solve problems that arise in mathematics and in other contexts;
- Apply and adapt a variety of appropriate strategies to solve problems;
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics
- Make decisions about units and scales that are appropriate for problem situations involving measurement
- Monitor and reflect on the process of mathematical problem solving.
- Select and use various types of reasoning and methods of proof.
- Organize and consolidate their mathematical thinking through communication.
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Materials/Resources

- County maps displaying all state maintained roads
- Highlighters, markers, crayons, glue, scissors, etc.
- Classroom set of rulers
- Tri-fold panel boards
- Computer lab reserved

Introduction: Setting Up the Mathematical Task

- Goal of the lesson.

  "In this lesson, groups or individuals will finalize their cost reduction proposals, type a formal proposal, and create a tri-fold panel board with their resulting data."

  (It is assumed that all students have submitted a science fair project in the past.)

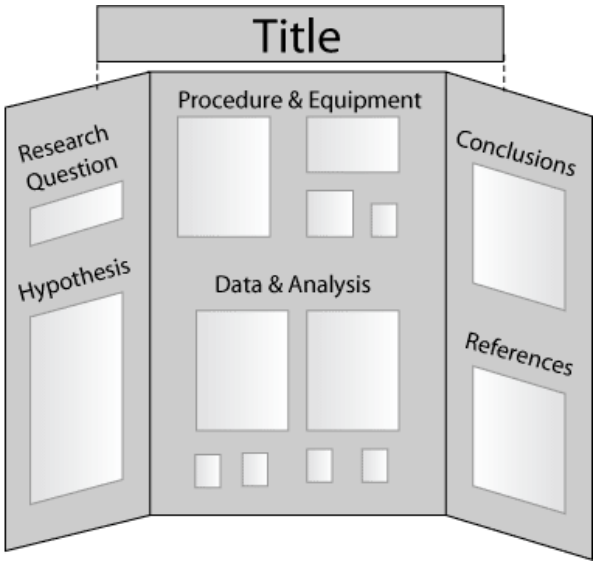
Time Line for the lesson: One 90-minute block

What	**Who**	**Amount of Time**
Revisit KWL Chart; begin to fill in "L" column with what we have learned about existing transportation costs	Students and Teacher	5 minutes
Groups or individuals will organize and compile their existing cost calculations and new proposals for cost reduction in paper format to turn in for a grade	Students	55 minutes
Groups or individuals will organize their information onto a tri-fold panel board (similar to a science fair project, to be presented to school board members and other county officials at a math fair)	Students	30 minutes

Expectations for Final Proposal Paper and Tri-fold Panel:

Paper: Format will be double-spaced, Times New Roman, 12 point font, printed front side only. Content will include accurate existing cost of transportation (from KWL chart), accurate proposed calculations, explanation and justification of cost reductions and total savings, and well thought out logistics of the proposal.

Tri-fold: The tri-fold panel will be designed according to the following picture (titles for sections may be amended as needed):



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Assessment:

Rubric for Tri-Fold Panel:

Excellent – 4 points	Good – 3 points	Average – 2 points	Poor – 1 point	0 points
Neat, organized, all elements included, eye-catching	Neat, organized, all elements included	Partially neat, partially organized, 1 element missing	Lacking neatness and organization, more than one element missing	Group or student chose not to participate

Rubric for proposed transportation calculations:

	Coding of existing bus routes	Mileage distance calculations	Fuel cost calculations	Calculation of Average cost per day of Annual Costs	Totals per day per bus	Grand total for high school
4	Legible, clearly defined, includes a key	Calculations are 93 to 100% accurate	Calculations are 93 to 100% accurate	Calculations are 93 to 100% accurate	Calculations are 93 to 100% accurate	Calculations are 93 to 100% accurate
3	Legible, somewhat defined, includes a key	Calculations are 85 to 92% accurate	Calculations are 85 to 92% accurate	Calculations are 85 to 92% accurate	Calculations are 85 to 92% accurate	Calculations are 85 to 92% accurate
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0	Not legible, not clearly defined, no key	Calculations are 68% or less accurate	Calculations are 68% or less accurate	Calculations are 68% or less accurate	Calculations are 68% or less accurate	Calculations are 68% or less accurate

Rubric for Final Proposal Paper

	Format of Paper	**Existing Cost of Transportation (from column L on KWL chart)**	**Cost of Proposed Transportation (to be graded on separate rubic)**	**Explanation and Justification of Cost Reduction and Total Savings**	**Logistics for Proposal**
4	Double-Spaced, Times New Roman, 12-Font, Printed on Front Only	Calculations are accurate	Included	Clearly articulated and justified	Thorough and well thought out
3				Clearly articulated but not fully justified	1 concern not addressed
2				Weak articulation, but fully justified	2 concerns not addressed
1				Weak articulation, weak justified	3 concerns not addressed
0	Instructed Format Not Followed	Inaccurate	Not included	Weak to little articulation and/or justification	More than 3 concerns not addressed; major concerns with logic

Extensions for this Unit:

- Students can present their final proposal paper and tri-fold panel in a math fair. Education officials (school board, administrators, superintendent, department of transportation) will be invited to attend and judge the entries.
- Local news officials may be invited to the math fair.
- The top three entries may present their proposals to the school board for a real life cost reduction experience.

- Students may conduct further research into best times for high-school students and how much sleep is required by high-school aged students.
- Students may find the constant of proportionality knowing that the total miles traveled varies directly with the number of gallons of gas and use this information to make predictions about future bus routes.